



# Vibration Institute



## Vibration Analysis Summary of Common Formulae

English Language  
US Customary and SI Units  
Rev Date: 2024-11-14

### Introduction:

The following pages contain a collection of equations, conversions, other vibration related information, and an instructional/example “bubble” style answer sheet. This information has been assembled primarily to aid examinees during the Vibration Institute’s ISO 18436-2 based certification examinations and may also have value as a reference.

Beyond the example “bubble” style answer sheet on the last page, this “Summary of Common Formulae” (commonly referred to as an Equation Sheet) may contain helpful information for VI VA certification examinees.

This “Summary of Common Formulae” is intended to be a resource for vibration analysts and may be included with Vibration Institute certification examinations. When received as a portion of a certification exam packet, it **MUST** remain with the packet and be placed into the completed exam envelope along with the exam, equation sheets, and bubble answer sheet.

*Note: personal copies of the “Summary of Common Formulae” shall **NOT** be used during a VI VA certification examination.*

The following sheets include:

- A: Units\*, Constants\* and Conversions\*
- B: Forces
- C: Motions
- D: Frequencies
- E: Signal Processing
- F: Vibration Response
- G: Stability Threshold
- H: Amplification Factor and Damping Ratio
- I: Material Properties\*
- J: Area Moments of Inertia
- K: Mass Moments of Inertia
- L: Cases of Stiffness Calculation
- Instructional/Example “Bubble” Style Answer Sheet

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Vibration Analyst: Category IV Equations

Revision: 2 2024-11-14 English/ US & SI

Approved: N. Denton

	US Customary Units	SI Units
<b>A</b>	<b>UNITS*, CONSTANTS* AND CONVERSIONS*</b>	
<b>①</b> <b>②</b> <b>③</b> <b>④</b>	length: foot, ft mass: slug, lbf s <sup>2</sup> /ft time: second, s force: pound force, lbf pressure: psi, lbf/in <sup>2</sup> acceleration: g, ft/s <sup>2</sup> or in/s <sup>2</sup> velocity: ft/s or in/s rotational speed: RPM, RPS, Hz, rad/s plane angle: degree, deg, ° full circle = 360° = 2π rad weight, W: lbf  pi, π = 3.142 standard acceleration due to gravity, g <sub>n</sub> = 32.17 ft/s <sup>2</sup> = 386.1 in/s <sup>2</sup> mass** = W/g <sub>n</sub>  1 lbf = 0.2248 N  1 foot (ft) = 12 inches (in) 1 inch (in) = 1000 mils 1 inch (in) = 0.02540 meter =25.4 mm  1 in <sup>2</sup> = 6.452 cm <sup>2</sup> 1 in <sup>3</sup> = 16.39 cm <sup>3</sup>  1 pound (lbf) = 16 ounces (oz) 1 ounce (oz)** = 28.35 grams (g)  1 psi = 6.895 kPa  1 lbf/in = 175.1 N/m = 0.1751 N/mm  *conversions and constants shown may be approximate, rounded, or stated for the field of vibration. **where acceleration due to gravity = g <sub>n</sub>	length: meter, m mass: kilogram, kg time: second, s force: Newton, N pressure, Pascal, 1 Pa = 1 N/m <sup>2</sup> acceleration: g, m/s <sup>2</sup> velocity: m/s rotational speed: RPM, RPS, Hz, rad/s plane angle: radian, rad full circle = 2π rad = 360° weight, W: N  pi, π = 3.142 standard acceleration due to gravity, g <sub>n</sub> = 9.807 m/s <sup>2</sup>  1 N = 4.448 lbf  1 m = 39.37 in = 1000 mm 1 mm = 0.03937 in 1 micron (μm) = 0.00003937 in = 0.0010 mm  1 cm <sup>2</sup> = 0.1550 in <sup>2</sup> 1 cm <sup>3</sup> = 0.0610 in <sup>3</sup>  1 kg = 2.205 lbm** 1 gram (g) = 0.0353 oz**  1 kPa = 0.1450 psi = 0.001 MPa  1 kN/m = 5.710 lbf/in = 1 N/mm  *conversions and constants shown may be approximate, rounded, or stated for the field of vibration. **where acceleration due to gravity = g <sub>n</sub>

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<b>B</b>	<b>FORCES</b>	
② ③ ④	<b>Mass Unbalance Force (lbf)</b>  $F = Me \left( \frac{2\pi N}{60} \right)^2$  M = W/g <sub>n</sub> , mass, lbf s <sup>2</sup> /in W = weight of rotor or balance weight, lbf g <sub>n</sub> = 386.1 in/s <sup>2</sup> e = rotor eccentricity or radius of balance weight, in N = rotational speed, rpm	<b>Mass Unbalance Force (Newtons)</b>  $F = Me \left( \frac{2\pi N}{60} \right)^2$  M = mass, kg e = rotor eccentricity or radius of balance mass, m N = rotational speed, rpm
② ③ ④	<b>Spring Force (lbf)</b>  $F = Kx$  K = stiffness of spring, lbf/in x = relative deflection, in	<b>Spring Force (Newtons)</b>  $F = Kx$  K = stiffness of spring, N/m x = relative deflection, m
③ ④	<b>Damping Force (lbf)</b>  $F = C\dot{x}$  C = damping constant, lbf s/in $\dot{x}$ = relative velocity, in/s	<b>Damping Force (Newtons)</b>  $F = C\dot{x}$  C = damping constant, N s/m $\dot{x}$ = relative velocity, m/s
③ ④	<b>Inertia Force (lbf)</b>  $F = M\ddot{x}$  M = mass, lbf s <sup>2</sup> /in $\ddot{x}$ = acceleration, in/s <sup>2</sup>	<b>Inertia Force (Newtons)</b>  $F = M\ddot{x}$  M = mass, kg $\ddot{x}$ = acceleration, m/s <sup>2</sup>

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<b>C</b>		<b>MOTIONS</b>	
<b>1</b> <b>2</b> <b>3</b> <b>4</b>	<b>Acceleration</b>  $A = V(2\pi f)$  A = acceleration, in/s <sup>2</sup> V = velocity, in/s f = frequency in Hz (CPS)	<b>Acceleration</b>  $A = \frac{V(2\pi f)}{1000}$  A = acceleration, m/s <sup>2</sup> V = velocity, mm/s f = frequency in Hz (CPS)	
	<b>1</b> <b>2</b> <b>3</b> <b>4</b>	<b>Velocity (in/s)</b>  $V = D(2\pi f)$  D = peak displacement, in f = frequency, cycle/s (cps or Hz)	<b>Velocity (mm/s)</b>  $V = D(2\pi f)$  D = peak displacement, mm f = frequency, cycle/s (cps or Hz)
	<b>1</b> <b>2</b> <b>3</b> <b>4</b>	<b>Displacement (in peak)</b>  $D = \frac{V}{2\pi f}$  $D = \frac{A}{(2\pi f)^2}$  V = velocity, in/s f = frequency in Hz (CPS) A = acceleration, in/s <sup>2</sup>	<b>Displacement (mm peak)</b>  $D = \frac{V}{2\pi f}$  $D = \frac{1000 A}{(2\pi f)^2}$  V = velocity, mm/s f = frequency in Hz (CPS) A = acceleration, m/s <sup>2</sup>
<b>D</b>		<b>FREQUENCIES</b>	
<b>2</b> <b>3</b> <b>4</b>	<b>Natural Frequency – Classical</b>  $f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$  k = stiffness, lbf/in m = W/g <sub>n</sub> W = weight, lbf g <sub>n</sub> = 386.1 in/s <sup>2</sup> f <sub>n</sub> = natural frequency of a single-degree-of-freedom system, Hz	<b>Natural Frequency - Classical</b>  $f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$  k = stiffness, N/m m = mass, kg  f <sub>n</sub> = natural frequency of a single-degree-of-freedom system, Hz	

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<p>② ③ ④</p>	<p><b>Frequency Unit Conversion</b></p> $\omega = 2\pi f$ <p><math>\omega</math> = frequency, rad/s  <math>f</math> = frequency, Hz or CPS</p>	<p><b>Frequency Unit Conversion</b></p> $\omega = 2\pi f$ <p><math>\omega</math> = frequency, rad/s  <math>f</math> = frequency, Hz or CPS</p>
<p>② ③ ④</p>	<p><b>Roll Rotational Speed</b></p> $\Omega = \frac{V}{5\pi D}$ <p><math>V</math> = web velocity, ft/min  <math>D</math> = roll diameter, in  <math>\Omega</math> = rotational speed, RPS</p>	<p><b>Roll Rotational Speed</b></p> $\Omega = \frac{16.66 V}{\pi D}$ <p><math>V</math> = web velocity, m/min  <math>D</math> = roll diameter, mm  <math>\Omega</math> = rotational speed, Hz</p>
<p>③ ④</p>	<p><b>Natural Frequency - Static Deflection*</b></p> $f_n = \frac{1}{2\pi} \sqrt{\frac{g_n}{\delta_{st}}}$ <p><math>g_n = 386.1 \text{ in/s}^2</math>  <math>\delta_{st}</math> = static deflection, in  <math>f_n</math> = natural frequency of single-degree-of-freedom system, Hz  *Only valid for linear spring rate and where the static deflection is in-line with gravity.</p>	<p><b>Natural Frequency - Static Deflection*</b></p> $f_n = \frac{1}{2\pi} \sqrt{\frac{g_n}{\delta_{st}}}$ <p><math>g_n = 9.807 \text{ m/s}^2</math>  <math>\delta_{st}</math> = static deflection, m  <math>f_n</math> = natural frequency of single-degree-of-freedom system, Hz  *Only valid for linear spring rate and where the static deflection is in-line with gravity.</p>

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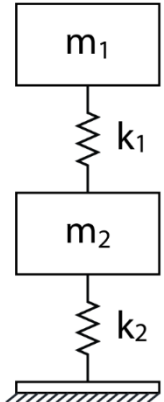
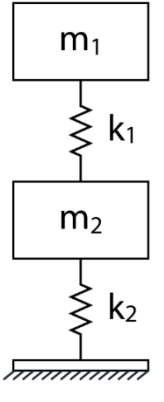
	<b>Bearing Frequencies</b>	<b>Bearing Frequencies</b>
2 3 4	$FTF_i = \left(\frac{\Omega}{2}\right) \left[1 - \left(\frac{B}{P}\right) \cos(CA)\right]$ $FTF_o = \left(\frac{\Omega}{2}\right) \left[1 + \left(\frac{B}{P}\right) \cos(CA)\right]$ $BPF_i = \left(\frac{N}{2}\right) \Omega \left[1 + \left(\frac{B}{P}\right) \cos(CA)\right]$ $BPFO = \left(\frac{N}{2}\right) \Omega \left[1 - \left(\frac{B}{P}\right) \cos(CA)\right]$ $BSF = \left(\frac{P}{2B}\right) \Omega \left[1 - \left(\frac{B}{P}\right)^2 \cos^2(CA)\right]$ <p>           FTF = fundamental train frequency            FTF<sub>i</sub> = inner race rotating            FTF<sub>o</sub> = outer race rotating            BPF<sub>i</sub> = ball pass frequency, inner race            BPFO = ball pass frequency, outer race            BSF = ball spin frequency            CA = contact angle, deg            Ω = rotating race speed (Hz or RPM)            N = number of rolling elements per row            P = pitch diameter, in            B = ball or roller diameter, in         </p>	$FTF_i = \left(\frac{\Omega}{2}\right) \left[1 - \left(\frac{B}{P}\right) \cos(CA)\right]$ $FTF_o = \left(\frac{\Omega}{2}\right) \left[1 + \left(\frac{B}{P}\right) \cos(CA)\right]$ $BPF_i = \left(\frac{N}{2}\right) \Omega \left[1 + \left(\frac{B}{P}\right) \cos(CA)\right]$ $BPFO = \left(\frac{N}{2}\right) \Omega \left[1 - \left(\frac{B}{P}\right) \cos(CA)\right]$ $BSF = \left(\frac{P}{2B}\right) \Omega \left[1 - \left(\frac{B}{P}\right)^2 \cos^2(CA)\right]$ <p>           FTF = fundamental train frequency            FTF<sub>i</sub> = inner race rotating            FTF<sub>o</sub> = outer race rotating            BPF<sub>i</sub> = ball pass frequency, inner race            BPFO = ball pass frequency, outer race            BSF = ball spin frequency            CA = contact angle, deg            Ω = rotating race speed (Hz or RPM)            N = number of rolling elements per row            P = pitch diameter, mm            B = ball or roller diameter, mm         </p>

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<p>① ② ③ ④</p>	<p><b>General Guidelines For Bearing Frequencies*</b></p> <p>BPFO = 0.41 x N x Ω          BPF1 = 0.59 x N x Ω          FTF<sub>i</sub> = 0.41 x Ω**          FTF<sub>o</sub> = 0.59 x Ω**          BSF = 0.22 x N x Ω</p> <p>N = number of rolling elements per row          Ω = speed units of RPM or Hz</p> <p>*for use in F<sub>Max</sub> selection ONLY.          ** FTF<sub>i</sub> applies when inner race rotates;          FTF<sub>o</sub> applies when outer race rotates</p>	<p><b>General Guidelines For Bearing Frequencies*</b></p> <p>BPFO = 0.41 x N x Ω          BPF1 = 0.59 x N x Ω          FTF<sub>i</sub> = 0.41 x Ω          FTF<sub>o</sub> = 0.59 x Ω          BSF = 0.22 x N x Ω</p> <p>N = number of rolling elements per row          Ω = speed units of RPM or Hz</p> <p>*for use in F<sub>Max</sub> selection ONLY.          ** FTF<sub>i</sub> applies when inner race rotates;          FTF<sub>o</sub> applies when outer race rotates</p>
<p>④</p>	<p><b>Natural Frequencies – Undamped 2 Degree of Freedom</b></p> $\omega^2 = \frac{k_2 + k_1}{2m_2} + \frac{k_1}{2m_1} \pm \sqrt{\frac{1}{4} \left[ \frac{k_2 + k_1}{m_2} + \frac{k_1}{m_1} \right]^2 - \frac{k_2 k_1}{m_2 m_1}}$ $f_{1,2} = \frac{1}{2\pi} \omega_{1,2}$ <p>k<sub>1</sub>, k<sub>2</sub> = stiffness, lbf/in          m<sub>1</sub>, m<sub>2</sub> = mass, lbf-s<sup>2</sup>/in          ω = natural frequency, rad/s</p> 	<p><b>Natural Frequencies – Undamped 2 Degree of Freedom</b></p> $\omega^2 = \frac{k_2 + k_1}{2m_2} + \frac{k_1}{2m_1} \pm \sqrt{\frac{1}{4} \left[ \frac{k_2 + k_1}{m_2} + \frac{k_1}{m_1} \right]^2 - \frac{k_2 k_1}{m_2 m_1}}$ $f_{1,2} = \frac{1}{2\pi} \omega_{1,2}$ <p>k<sub>1</sub>, k<sub>2</sub> = stiffness, N/m          m<sub>1</sub>, m<sub>2</sub> = mass, kg          ω = natural frequency, rad/s</p> 

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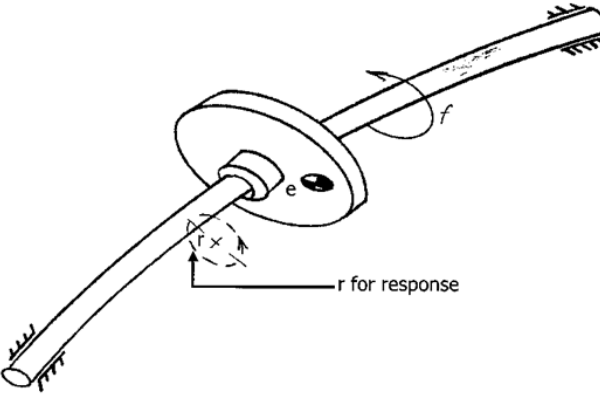
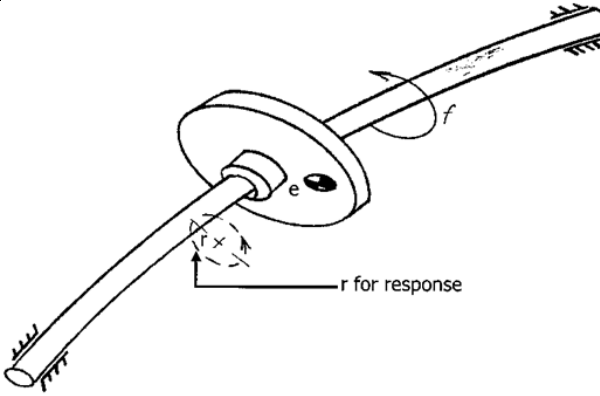
E	SIGNAL PROCESSING	
① ② ③ ④	<b>Peak vs. RMS*</b>  peak = 1.414 rms  *Applies to single frequency sine waveforms only.	<b>Peak vs. RMS*</b>  peak = 1.414 rms  *Applies to single frequency sine waveforms only.
① ② ③ ④	<b>Default Frequency Spans</b>  Operating Speed $F_{MAX} \geq 10 \times \text{RPM}$  Rolling Element Bearings $F_{MAX} \geq 10 \times \text{BPFI}$  Fluid Film Bearings $F_{MAX} \geq 10 \times \text{RPM}$  Vane/Blade Pass $F_{MAX} \geq 3 \times \# \text{ Vanes/Blades} \times \text{RPM}$  Electrical $F_{MAX} \geq 3 \times 2X \text{ Line Frequency}$  Gear Mesh $F_{MAX} \geq 3 \times \text{Gear Mesh Frequency}$  RPM = rotational speed  $F_{MAX}$ = maximum frequency	<b>Default Frequency Spans</b>  Operating Speed $F_{MAX} \geq 10 \times \text{RPM}$  Rolling Element Bearings $F_{MAX} \geq 10 \times \text{BPFI}$  Fluid Film Bearings $F_{MAX} \geq 10 \times \text{RPM}$  Vane/Blade Pass $F_{MAX} \geq 3 \times \# \text{ Vanes/Blades} \times \text{RPM}$  Electrical $F_{MAX} \geq 3 \times 2X \text{ Line Frequency}$  Gear Mesh $F_{MAX} \geq 3 \times \text{Gear Mesh Frequency}$  RPM = rotational speed  $F_{MAX}$ = maximum frequency
② ③ ④	<b>Frequency Resolution*</b>  Frequency resolution = (frequency span x window noise factor x 2) / # of FFT lines  Window Noise Factor: 1.0 for uniform, rectangular, “no window” 1.5 for Hanning window 3.8 for flat top window  *ability to resolve closely spaced signals	<b>Frequency Resolution*</b>  Frequency resolution = (frequency span x window noise factor x 2) / (# of FFT lines)  Window Noise Factor: 1.0 for uniform, rectangular, “no window” 1.5 for Hanning window 3.8 for flat top window  *ability to resolve closely spaced signals
② ③ ④	<b>Block Data Acquisition Time (DAT)*</b>  DAT = # FFT lines/frequency span  *Applicable when Nyquist Factor = 2.56	<b>Block Data Acquisition Time (DAT)*</b>  DAT = # FFT lines/frequency span  *Applicable when Nyquist Factor = 2.56

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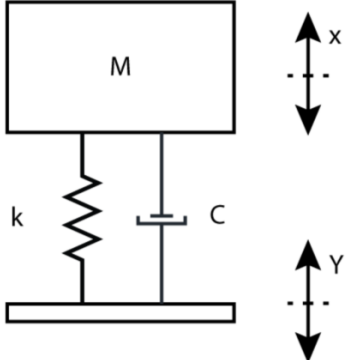
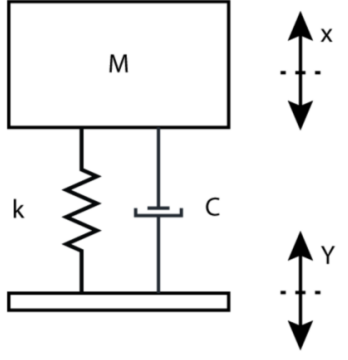
<p>③</p> <p>④</p>	<p><b>Dynamic Range</b></p> $DR = 20 \log \frac{V_m}{V_r}$ $\frac{V_m}{V_r} = 10^{\frac{dB}{20}}$ <p><math>V_m</math> = voltage measured  <math>V_r</math> = voltage reference  DR = dynamic range, dB (decibels)</p>	<p><b>Dynamic Range</b></p> $DR = 20 \log \frac{V_m}{V_r}$ $\frac{V_m}{V_r} = 10^{\frac{dB}{20}}$ <p><math>V_m</math> = voltage measured  <math>V_r</math> = voltage reference  DR = dynamic range, dB (decibels)</p>
<p><b>F VIBRATION RESPONSE</b></p>		
<p>④</p>	<p><b>Rotor Vibration Response</b></p>  $r = \frac{e \left( \frac{f}{f_n} \right)^2}{\sqrt{\left[ 1 - \left( \frac{f}{f_n} \right)^2 \right]^2 + \left[ 2\zeta \frac{f}{f_n} \right]^2}}$ <p><math>r</math> = rotor vibration response, in peak  <math>e</math> = eccentricity of mass, in  <math>f</math> = forcing frequency, Hz  <math>f_n</math> = natural frequency, Hz  <math>\zeta</math> = damping ratio, <math>C/C_c</math>  <math>C</math> = damping constant, lbf s/in  <math>C_c</math> = critical damping = <math>2 m \omega_n</math>  <math>m</math> = mass, lbf s<sup>2</sup>/in</p> $\omega_n = \sqrt{\frac{k}{m}}, \text{ rad/s}$	<p><b>Rotor Vibration Response</b></p>  $r = \frac{e \left( \frac{f}{f_n} \right)^2}{\sqrt{\left[ 1 - \left( \frac{f}{f_n} \right)^2 \right]^2 + \left[ 2\zeta \frac{f}{f_n} \right]^2}}$ <p><math>r</math> = rotor vibration response, mm peak  <math>e</math> = eccentricity of mass, mm  <math>f</math> = forcing frequency, Hz  <math>f_n</math> = natural frequency, Hz  <math>\zeta</math> = damping ratio, <math>C/C_c</math>  <math>C</math> = damping constant, N s/m  <math>C_c</math> = critical damping = <math>2 m \omega_n</math>  <math>m</math> = mass, N s<sup>2</sup>/m (= kg)</p> $\omega_n = \sqrt{\frac{k}{m}}, \text{ rad/s}$

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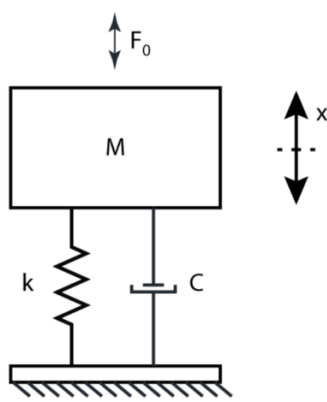
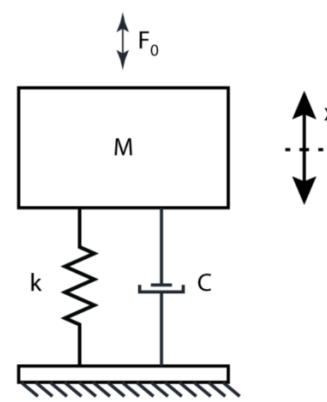
<p>④</p>	<p><b>Phase</b></p> $\tan \phi = \frac{2\zeta \frac{f}{f_n}}{1 - \left(\frac{f}{f_n}\right)^2}$ <p><math>\phi</math> = phase, deg  f = forcing frequency, Hz  f<sub>n</sub> = natural frequency, Hz</p>	<p><b>Phase</b></p> $\tan \phi = \frac{2\zeta \frac{f}{f_n}}{1 - \left(\frac{f}{f_n}\right)^2}$ <p><math>\phi</math> = phase, deg  f = forcing frequency, Hz  f<sub>n</sub> = natural frequency, Hz</p>
<p>④</p>	<p><b>Transmissibility</b></p>  <p>The diagram shows a mass M supported by a spring with stiffness k and a damper with constant C. The base of the system moves vertically with displacement Y, and the mass moves with displacement X.</p> $\frac{X}{Y} = \frac{F_{TR}}{F_0} = \frac{\sqrt{1 + \left[2\zeta \frac{f}{f_n}\right]^2}}{\sqrt{\left[1 - \left(\frac{f}{f_n}\right)^2\right]^2 + \left[2\zeta \frac{f}{f_n}\right]^2}}$ <p>F<sub>TR</sub> = transmitted force, lbf  F<sub>0</sub> = exciting force, lbf  f = forcing frequency, Hz  f<sub>n</sub> = natural frequency, Hz  ω<sub>n</sub> = natural frequency, rad/s  X = mass motion, in peak  Y = base motion, in peak  ζ = damping ratio = C/C<sub>c</sub>  k = spring stiffness, lbf/in  C = damping constant, lbf s/in  C<sub>c</sub> = 2mω<sub>n</sub> = critical damping, lbf s/in  M = mass, lbf-s<sup>2</sup>/in</p>	<p><b>Transmissibility</b></p>  <p>The diagram shows a mass M supported by a spring with stiffness k and a damper with constant C. The base of the system moves vertically with displacement Y, and the mass moves with displacement X.</p> $\frac{X}{Y} = \frac{F_{TR}}{F_0} = \frac{\sqrt{1 + \left[2\zeta \frac{f}{f_n}\right]^2}}{\sqrt{\left[1 - \left(\frac{f}{f_n}\right)^2\right]^2 + \left[2\zeta \frac{f}{f_n}\right]^2}}$ <p>F<sub>TR</sub> = transmitted force, N  F<sub>0</sub> = exciting force, N  f = forcing frequency, Hz  f<sub>n</sub> = natural frequency, Hz  ω<sub>n</sub> = natural frequency, rad/s  X = mass motion, mm peak  Y = base motion, mm peak  ζ = damping ratio = C/C<sub>c</sub>  k = spring stiffness, N/m  C = damping constant, N s/m  C<sub>c</sub> = 2mω<sub>n</sub> = critical damping, N s/m  M = mass, kg</p>

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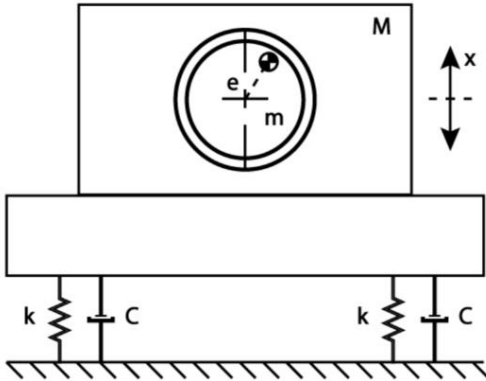
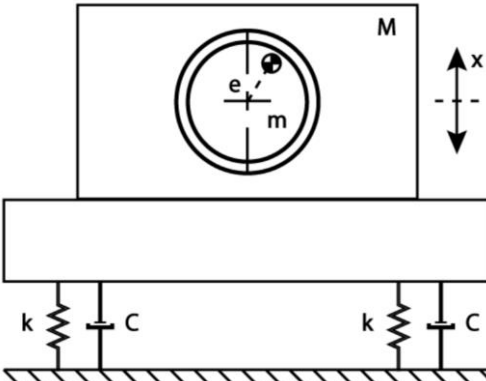
4	<p style="text-align: center;"><b>Forced Vibration Response</b></p>  $X = \frac{\frac{F_0}{k}}{\sqrt{\left[1 - \left(\frac{f}{f_n}\right)^2\right]^2 + \left[2\zeta \frac{f}{f_n}\right]^2}}$ <p> <math>F_0</math> = exciting force, lbf  <math>f</math> = forcing frequency, Hz  <math>f_n</math> = natural frequency, Hz  <math>\omega_n</math> = natural frequency, rad/s  <math>X</math> = mass motion, inches peak  <math>\zeta</math> = damping ratio = <math>C/C_c</math>  <math>k</math> = spring stiffness, lbf/in  <math>C</math> = damping constant, lbf s/in  <math>C_c = 2M\omega_n</math> = critical damping, lbf s/in  <math>M</math> = mass, lbf s<sup>2</sup>/in </p>	<p style="text-align: center;"><b>Forced Vibration Response</b></p>  $X = \frac{\frac{F_0}{k}}{\sqrt{\left[1 - \left(\frac{f}{f_n}\right)^2\right]^2 + \left[2\zeta \frac{f}{f_n}\right]^2}}$ <p> <math>F_0</math> = exciting force, N  <math>f</math> = forcing frequency, Hz  <math>f_n</math> = natural frequency, Hz  <math>\omega_n</math> = natural frequency, rad/s  <math>X</math> = mass motion, mm peak  <math>\zeta</math> = damping ratio = <math>C/C_c</math>  <math>k</math> = spring stiffness, N/m  <math>C</math> = damping constant, N s/m  <math>C_c = 2M\omega_n</math> = critical damping, N s/m  <math>M</math> = mass, kg </p>
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<p>④</p>	<p><b>Mass Unbalance Induced Casing Response Ratio</b></p>  $\frac{MX}{me} = \frac{\left(\frac{f}{f_n}\right)^2}{\sqrt{\left[1 - \left(\frac{f}{f_n}\right)^2\right]^2 + \left[2\zeta\frac{f}{f_n}\right]^2}}$ <p> M = total mass, lbf s<sup>2</sup>/in  m = rotor mass, lbf s<sup>2</sup>/in  X = casing motion, inches peak  e = rotor eccentricity, inches  f = forcing frequency, Hz  f<sub>n</sub> = natural frequency, Hz  ω<sub>n</sub> = natural frequency, rad/s  ζ = damping ratio = C/C<sub>c</sub>  k = spring stiffness, lbf/in  C = damping constant, lbf s/in  C<sub>c</sub> = 2Mω<sub>n</sub> = critical damping, lbf s/in </p>	<p><b>Mass Unbalance Induced Casing Response Ratio</b></p>  $\frac{MX}{me} = \frac{\left(\frac{f}{f_n}\right)^2}{\sqrt{\left[1 - \left(\frac{f}{f_n}\right)^2\right]^2 + \left[2\zeta\frac{f}{f_n}\right]^2}}$ <p> M = total mass, kg  m = rotor mass, kg  X = casing motion, mm peak  e = rotor eccentricity, mm  f = forcing frequency, Hz  f<sub>n</sub> = natural frequency, Hz  ω<sub>n</sub> = natural frequency, rad/s  ζ = damping ratio = C/C<sub>c</sub>  k = spring stiffness, N/m  C = damping constant, N s/m  C<sub>c</sub> = 2Mω<sub>n</sub> = critical damping, N s/m </p>
<p><b>G Fluid Film Bearings</b></p>		
<p>④</p>	<p><b>Stability Threshold</b></p> $\omega_s = \omega_o \sqrt{\frac{c}{g_n}}$ <p> ω<sub>s</sub> = dimensionless stability threshold  ω<sub>o</sub> = journal operating speed, rad/s  c = bearing radial clearance, in  g<sub>n</sub> = standard gravity, 386.1 in/s<sup>2</sup> </p>	<p><b>Stability Threshold</b></p> $\omega_s = \omega_o \sqrt{\frac{c}{g_n}}$ <p> ω<sub>s</sub> = dimensionless stability threshold  ω<sub>o</sub> = journal operating speed, rad/s  c = bearing radial clearance, m  g<sub>n</sub> = standard gravity, 9.807 m/s<sup>2</sup> </p>

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④	<p>Sommerfeld Number</p> $S = \frac{\mu NLD}{W} \left(\frac{R}{C}\right)^2$ <p>S = Sommerfeld number  <math>\mu</math> = oil viscosity, lbf s/in<sup>2</sup>  N = shaft speed, RPS  L = bearing length, in  D = journal diameter, in  W = load on the bearing, lbf  R = radius of the journal, in  C = bearing radial clearance, in</p>	<p>Sommerfeld Number</p> $S = \frac{\mu NLD}{W} \left(\frac{R}{C}\right)^2$ <p>S = Sommerfeld number  <math>\mu</math> = oil viscosity, N s/mm<sup>2</sup>  N = shaft speed, RPS  L = bearing length, mm  D = journal diameter, mm  W = load on the bearing, N  R = radius of the journal, mm  C = bearing radial clearance, mm</p>
<b>H AMPLIFICATION FACTOR AND DAMPING RATIO</b>		
④	<p><b>Half Power Method</b></p> $AF = \frac{N_{CR}}{N_2 - N_1}$ <p>AF = amplification factor (at resonance)  N<sub>CR</sub> = critical speed, RPM  N<sub>2</sub>, N<sub>1</sub> = half power points, RPM</p>	<p><b>Half Power Method</b></p> $AF = \frac{N_{CR}}{N_2 - N_1}$ <p>AF = amplification factor (at resonance)  N<sub>CR</sub> = critical speed, Hz  N<sub>2</sub>, N<sub>1</sub> = half power points, Hz</p>
④	<p><b>Phase Change Method</b></p> $AF = \frac{\pi N_{CR}}{360} \frac{\Delta\phi}{\Delta N}$ <p>AF = amplification factor (at resonance)  N<sub>CR</sub> = critical speed, RPM  <math>\Delta\phi</math> = phase change, degrees  <math>\Delta N</math> = speed change, RPM</p>	<p><b>Phase Change Method</b></p> $AF = \frac{\pi N_{CR}}{360} \frac{\Delta\phi}{\Delta N}$ <p>AF = amplification factor (at resonance)  N<sub>CR</sub> = critical speed, Hz  <math>\Delta\phi</math> = phase change, degrees  <math>\Delta N</math> = speed change, Hz</p>

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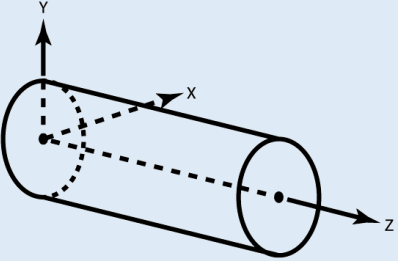
<p>④</p>	<p><b>Time Domain Method</b></p> $\delta = \frac{1}{n} \ln \frac{X_o}{X_n} = \text{log decrement}$ $\zeta = \frac{C}{C_c} \approx \frac{\delta}{2\pi} \approx \frac{1}{2AF} = \text{damping ratio}$ <p>n = number of cycles</p> $\frac{X_o}{X_n} = \text{amplitude ratio in n cycles}$ <p>C = damping constant, lbf s/in  <math>C_c = 2M\omega_n = \text{critical damping, lbf s/in}</math>  AF = amplification factor  M = system mass, lbf s<sup>2</sup>/in  <math>\omega_n = \text{system natural frequency, rad/s}</math></p>	<p><b>Time Domain Method</b></p> $\delta = \frac{1}{n} \ln \frac{X_o}{X_n} = \text{log decrement}$ $\zeta = \frac{C}{C_c} \approx \frac{\delta}{2\pi} \approx \frac{1}{2AF} = \text{damping ratio}$ <p>n = number of cycles</p> $\frac{X_o}{X_n} = \text{amplitude ratio in n cycles}$ <p>C = damping constant, N s/m  <math>C_c = 2M\omega_n = \text{critical damping, N s/m}</math>  AF = amplification factor  M = system mass, N s<sup>2</sup>/m  <math>\omega_n = \text{system natural frequency, rad/s}</math></p>
<p>④</p>	<p><b>Dual Constant/Dual Channel Method</b></p> <p>Damping Calculation:</p> $AF = \frac{\left(\frac{f_a}{f_b}\right)^2 + 1}{\left(\frac{f_a}{f_b}\right)^2 - 1}$ <p>f<sub>a</sub> = frequency above resonance, where the real or imaginary part of the transfer function reaches a peak  f<sub>b</sub> = frequency below resonance, where the real or imaginary part of the transfer function reaches a peak of opposite sign  AF = amplification factor (at resonance)</p> $AF \approx \frac{1}{2\frac{C}{C_c}} \approx \frac{1}{(2\zeta)}$ <p>C = damping constant, lbf s/in  <math>C_c = 2M\omega_n = \text{critical damping, lbf s/in}</math>  AF = amplification factor  M = system mass, lbf s<sup>2</sup>/in  <math>\omega_n = \text{system natural frequency, } \frac{\text{rad}}{\text{s}}</math></p>	<p><b>Dual Constant/Dual Channel Method</b></p> <p>Damping Calculation:</p> $AF = \frac{\left(\frac{f_a}{f_b}\right)^2 + 1}{\left(\frac{f_a}{f_b}\right)^2 - 1}$ <p>f<sub>a</sub> = frequency above resonance, where the real or imaginary part of the transfer function reaches a peak  f<sub>b</sub> = frequency below resonance, where the real or imaginary part of the transfer function reaches a peak of opposite sign  AF = amplification factor (at resonance)</p> $AF \approx \frac{1}{2\frac{C}{C_c}} \approx \frac{1}{(2\zeta)}$ <p>C = damping constant, N s/m  <math>C_c = 2M\omega_n = \text{critical damping, N s/m}</math>  AF = amplification factor  M = system mass, N s<sup>2</sup>/m  <math>\omega_n = \text{system natural frequency, } \frac{\text{rad}}{\text{s}}</math></p>

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<b>I MATERIAL PROPERTIES*</b>		
<b>4</b>	<p>E = Modulus of Elasticity</p> <p>Aluminum      10.0 x 10<sup>6</sup> lbf/in<sup>2</sup></p> <p>Ductile Iron    24.5 x 10<sup>6</sup> lbf/in<sup>2</sup></p> <p>Steel            29.0 x 10<sup>6</sup> lbf/in<sup>2</sup></p> <p>Titanium        14.9 x 10<sup>6</sup> lbf/in<sup>2</sup></p> <p>G = Modulus of Rigidity</p> <p>Aluminum      3.9 x 10<sup>6</sup> lbf/in<sup>2</sup></p> <p>Ductile Iron    9.3 x 10<sup>6</sup> lbf/in<sup>2</sup></p> <p>Steel            11.2 x 10<sup>6</sup> lbf/in<sup>2</sup></p> <p>Titanium        5.9 x 10<sup>6</sup> lbf/in<sup>2</sup></p>	<p>E = Modulus of Elasticity</p> <p>Aluminum      69 GPa</p> <p>Ductile Iron    169 GPa</p> <p>Steel            200 GPa</p> <p>Titanium        103 GPa</p> <p>G = Modulus of Rigidity (GPa = 10<sup>9</sup> Pa)</p> <p>Aluminum      27 GPa</p> <p>Ductile Iron    64 GPa</p> <p>Steel            77 GPa</p> <p>Titanium        41 GPa</p>
<b>4</b>	<p>Density</p> <p>Aluminum      0.0978 lbf/in<sup>3</sup></p> <p>Ductile Iron    0.2579 lbf/in<sup>3</sup></p> <p>Steel            0.2836 lbf/in<sup>3</sup></p> <p>Titanium        0.1620 lbf/in<sup>3</sup></p> <p>*constants shown may be approximate, rounded, or stated for the field of vibration.</p>	<p>Density</p> <p>Aluminum      2712 kg/m<sup>3</sup></p> <p>Ductile Iron    7300 kg/m<sup>3</sup></p> <p>Steel            7850 kg/m<sup>3</sup></p> <p>Titanium        4500 kg/m<sup>3</sup></p> <p>*constants shown may be approximate, rounded, or stated for the field of vibration.</p>
<b>J</b>	<p><b>AREA MOMENTS OF INERTIA</b></p>  <p><i>Figure also applies to Section K</i></p>	
<b>4</b>	<p><b>Bending Area Moment of Inertia</b></p> $I_x = I_y = \frac{\pi D^4}{64} \quad (\text{solid shaft})$ $I_x = I_y = \frac{\pi(D^4 - d^4)}{64} \quad (\text{hollow shaft})$ <p><math>I_x = I_y</math> = bending area moment of inertia of circular cross-section, in<sup>4</sup></p> <p><math>D</math> = outer diameter, in</p> <p><math>d</math> = inner diameter, in</p>	<p><b>Bending Area Moment of Inertia</b></p> $I_x = I_y = \frac{\pi D^4}{64} \quad (\text{solid shaft})$ $I_x = I_y = \frac{\pi(D^4 - d^4)}{64} \quad (\text{hollow shaft})$ <p><math>I_x = I_y</math> = bending area moment of inertia of circular cross-section, mm<sup>4</sup></p> <p><math>D</math> = outer diameter, mm</p> <p><math>d</math> = inner diameter, mm</p>

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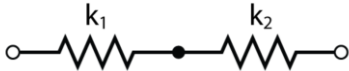
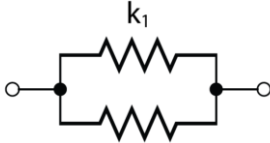
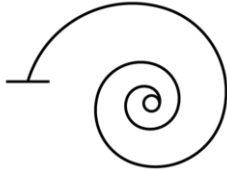
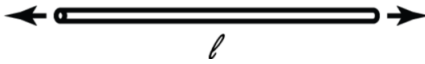
4	<p><b>Polar Area Moment of Inertia</b></p> $J = \frac{\pi D^4}{32} \quad (\text{solid shaft})$ $J = \frac{\pi(D^4 - d^4)}{32} \quad (\text{hollow shaft})$ <p>J = polar area moment of inertia of circular cross section, in<sup>4</sup>  D = outer diameter, in  d = inner diameter, in</p>	<p><b>Polar Area Moment of Inertia</b></p> $J = \frac{\pi D^4}{32} \quad (\text{solid shaft})$ $J = \frac{\pi(D^4 - d^4)}{32} \quad (\text{hollow shaft})$ <p>J = polar area moment of inertia of circular cross section, mm<sup>4</sup>  D = outer diameter, mm  d = inner diameter, mm</p>
<b>K MASS MOMENTS OF INERTIA</b>		
4	<p><b>Mass Moment of Inertia, Thin Disk</b></p> $I_x = I_y = \frac{MD^2}{16}$ <p>I<sub>x</sub> = I<sub>y</sub> = mass moment of inertia, lbf s<sup>2</sup> in  M = mass, lbf s<sup>2</sup>/in  D = diameter, in</p>	<p><b>Mass Moment of Inertia, Thin Disk</b></p> $I_x = I_y = \frac{MD^2}{16}$ <p>I<sub>x</sub> = I<sub>y</sub> = mass moment of inertia, kg m<sup>2</sup>  M = mass, kg  D = diameter, m</p>
4	<p><b>Polar Mass Moment of Inertia, Shaft</b></p> $J = \frac{MD^2}{8}$ <p>J = polar mass moment of inertia, lbf s<sup>2</sup> in  M = mass, lbf s<sup>2</sup>/in  D = diameter, in</p>	<p><b>Polar Mass Moment of Inertia, Shaft</b></p> $J = \frac{MD^2}{8}$ <p>J = polar mass moment of inertia, kg m<sup>2</sup>  M = mass, kg  D = diameter, m</p>

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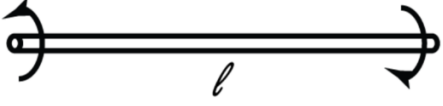

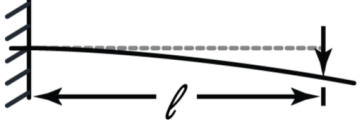
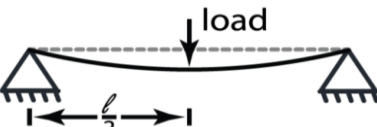
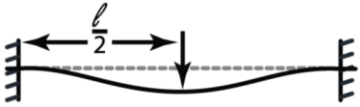
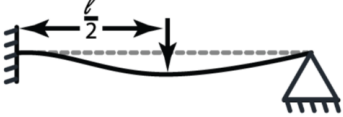
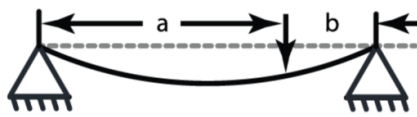
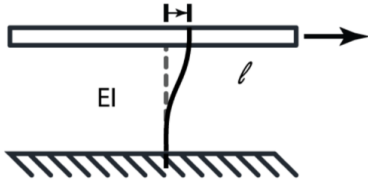
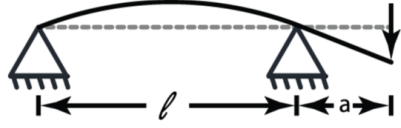

<b>L</b>		<b>CASES OF STIFFNESS CALCULATION</b>	
		<b>All cases shown apply to both US Customary and SI systems</b>	
<b>4</b>	<p><b>Common Parameters for US Customary Usage of Case formulas</b></p> <p>k = stiffness, lbf/in  <math>k_t</math> = torsional stiffness, lbf in/rad  I = bending area moment of inertia of circular cross-section, in<sup>4</sup>  J = polar area moment of inertia of circular cross section, in<sup>4</sup>  D = outer diameter, in  d = inner diameter, in  d = rod diameter of coil, in  A = cross sectional area, in<sup>2</sup>  n = number of turns, count  l = total length, in  E = Modulus of Elasticity, lbf/in<sup>2</sup>  G = Modulus of Rigidity, lbf/in<sup>2</sup></p>	<b>4</b>	<p><b>Common Parameters for SI Usage of Case formulas</b></p> <p>k = stiffness, N/m  <math>k_t</math> = torsional stiffness, N m/rad  I = bending area moment of inertia of circular cross-section, m<sup>4</sup>  J = polar area moment of inertia of circular cross section, m<sup>4</sup>  D = outer diameter, m  d = inner diameter, m  d = rod diameter of coil, m  A = cross sectional area, m<sup>2</sup>  n = number of turns, count  l = total length, m  E = Modulus of Elasticity, Pa  G = Modulus of Rigidity, Pa</p>
<b>4</b>	<p><b>Case A: Springs in series</b></p>  $k = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}}$	<b>4</b>	<p><b>Case B: Springs In parallel</b></p>  $k = k_1 + k_2$
<b>4</b>	<p><b>Case C:</b></p>  $k_t = \frac{EI}{\ell}$	<b>4</b>	<p><b>Case D:</b></p>  $k = \frac{EA}{\ell}$

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<p>④</p>	<p>Case E:</p>  $k_t = \frac{GJ}{\ell}$	<p>Case F:</p>  $k = \frac{Gd^4}{64nR^3}$
<p>④</p>	<p>Case G:</p>  $k = \frac{3EI}{\ell^3}$	<p>Case H:</p>  $k = \frac{48EI}{\ell^3}$
<p>④</p>	<p>Case I:</p>  $k = \frac{192EI}{\ell^3}$	<p>Case J:</p>  $k = \frac{768EI}{7\ell^3}$
<p>④</p>	<p>Case K:</p>  $k = \frac{3EI\ell}{a^2b^2}$ $\ell = a + b$	<p>Case L:</p>  $k = \frac{12EI}{\ell^3}$
<p>④</p>	<p>Case M:</p>  $k = \frac{3EI}{(\ell+a)a^2}$	<p>Case N:</p>  $k = \frac{12EI}{a^2(3\ell+4a)}$

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# Instructional / Example "Bubble" style answer sheet



**DATE OF BIRTH**  
 Month: JAN FEB MAR APR MAY JUN JUL AUG SEP OCT NOV DEC  
 Day: 1 2 3 4 5 6 7 8 9 10 11 12  
 Year: 1 2 3 4 5 6 7 8

**I.D. NUMBER**  
 1 2 3 4 5 6 7 8

**IMPORTANT**  
 - ERASE COMPLETELY TO CHANGE  
 • EXAMPLE: A B C D E  
 • ERASE COMPLETELY TO CHANGE

**GENERAL PURPOSE BUBBLE SHEET**

**LAST NAME**  
 JOHNSON

**FIRST NAME**  
 JAMES

**MIDDLE INITIAL**  
 L

**CODES**  
 5 2 1 8

**DATE OF EXAM** (points to Date of Birth section)

**ID number from label on page 1** (points to I.D. Number section)

**Last name (first 12 characters)** (points to JOHNSON)

**First name (first 8 characters)** (points to JAMES)

**Middle Initial (blank if none)** (points to L)

**"CODES" from your exam's upper left page header.** (points to 5 2 1 8)

**All entries must have the appropriate "marks" filled in per the instructions in order for the information above and the answers to the exam questions read accurately for scoring.**

**As an example, answers for questions 1 thru 7 have been marked by filling in the appropriate rectangle.**

1 - A    5 - C  
 2 - B    6 - B  
 3 - C    7 - A  
 4 - D

**If needed, the response marks for questions 101 thru 250 are on the reverse side.**

QUESTIONS 101 THROUGH 250 CONTINUED ON THE OTHER SIDE

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