Vibration Institute

Vibration Analysis
Summary of Common Formulae

Please send comments to ndenton@vi-institute.org by March 15, 2024

English Language
US Customary and SI Units
Rev: 2024-01-11 DRAFT

Introduction:

The following pages contain a collection of equations, conversions, other vibration related information, and an instructional/example “bubble” style answer sheet. This information has been assembled primarily to aid examinees during the Vibration Institute’s ISO 18436-2 based certification examinations and may also have value as a reference.

Beyond the example “bubble” style answer sheet on the last page, this “Summary of Common Formulae” (commonly referred to as an Equation Sheet) may contain helpful information for VI VA certification examinees.

This “Summary of Common Formulae” is intended to be a resource for vibration analysts. This ***draft*** version is under consideration to replace the Summary of Common Formulae currently in the VI VA certification examination packets. When received as a portion of a certification exam packet, it MUST remain with the packet and be placed into the completed exam envelope along with the exam, equation sheets, and bubble answer sheet.

Note: personal copies of the “Summary of Common Formulae” shall NOT be used during a VI VA certification examination.

The following sheets include:

- A: Units*, Constants* and Conversions*
- B: Forces
- C: Motions
- D: Frequencies
- E: Signal Processing
- F: Vibration Response
- G: Stability Threshold
- H: Amplification Factor and Damping Ratio
- I: Material Properties*
- J: Area Moments of Inertia
- K: Mass Moments of Inertia
- L: Cases of Stiffness Calculation
- Instructional/Example “Bubble” Style Answer Sheet

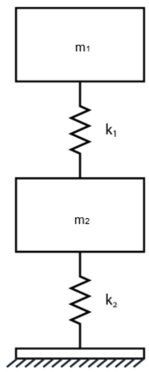
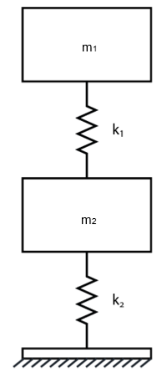
	US Customary Units	SI Units
Int. Ref	**A: UNITS*, CONSTANTS* AND CONVERSIONS***	
❶ ❷ ❸ ❹	length: foot, ft mass: slug, lbf s^2/ft time: second, s force: pound force, lbf pressure: psi, lbf/in^2 acceleration: g, ft/s^2 or in/s^2 velocity: v, ft/s or in/s plane angle: degree, deg, ° full circle = 360° = 2π rad weight, W: lbf pi, π = 3.142 standard gravity, g_n = 32.17 ft/s^2 = 386.1 in/s^2 mass** = W/g_n 1 lbf = 0.2248 N 1 foot (ft) = 12 inches (in) 1 inch (in) = 1000 mils 1 inch (in) = 0.02540 meter =25.4 mm 1 in^2 = 6.452 cm^2 1 in^3 = 16.39 cm^3 1 pound (lbf) = 16 ounces (oz) 1 ounce (oz)** = 28.35 grams (g) 1 psi = 6.895 kPa 1 lbf/in = 175.1 N/m = 0.1751 N/mm *conversions and constants shown may be approximate, rounded, or stated for the field of vibration. **where gravity = g_n	length: meter, m mass: kilogram, kg time: second, s force: Newton, N pressure, Pascal, 1 Pa = 1 N/m^2 acceleration: g, m/s^2 velocity: m/s plane angle: radian, rad full circle = 2π rad = 360° weight, W: N pi, π = 3.142 standard gravity, g_n = 9.807 m/s^2 1 N = 4.448 lbf 1 m = 39.37 in = 1000 mm 1 mm = 0.03937 in 1 micron (µm) = 0.00003937 in = 0.0010 mm 1 cm^2 = 0.1550 in^2 1 cm^3 = 0.0610 in^3 1 kg = 2.205 lbm** 1 gram (g) = 0.0530 oz** 1 kPa = 0.1450 psi = 0.001 MPa 1 kN/m = 5.710 lbf/in = 1 N/mm *conversions and constants shown may be approximate, rounded, or stated for the field of vibration. **where gravity = g_n

Int. Ref	B: FORCES	
❷ ❸ ❹	**Mass Unbalance Force (lbf)** $F = Me\left(\frac{2\pi N}{60}\right)^2$ $M = W/g_n$, mass, lbf s^2/in W = weight of rotor or balance weight, lbf e = rotor eccentricity or radius of balance weight, in g_n = standard gravity, 386.1 in/s^2 N = rotational speed, rpm	**Mass Unbalance Force (N)** $F = Me\left(\frac{2\pi N}{60}\right)^2$ M = mass, kg e = rotor eccentricity or radius of balance mass, m N = rotational speed, rpm
❷ ❸ ❹	**Spring Force (lbf)** $F = Kx$ K = stiffness of spring, lbf/in x = relative deflection, in	**Spring Force (N)** $F = Kx$ K = stiffness of spring, N/m x = relative deflection, m
❸ ❹	**Damping Force (lbf)** $F = C\dot{x}$ C = damping constant, lbf s/in $\dot{x}$ = relative velocity, in/s	**Damping Force (N)** $F = C\dot{x}$ C = damping constant, N s/m $\dot{x}$ = relative velocity, m/s
❸ ❹	**Inertia Force (lbf)** $F = M\ddot{x}$ M = mass, lbf s^2/in $\ddot{x}$ = acceleration, in/s^2	**Inertia Force (N)** $F = M\ddot{x}$ M = mass, kg $\ddot{x}$ = acceleration, m/s^2

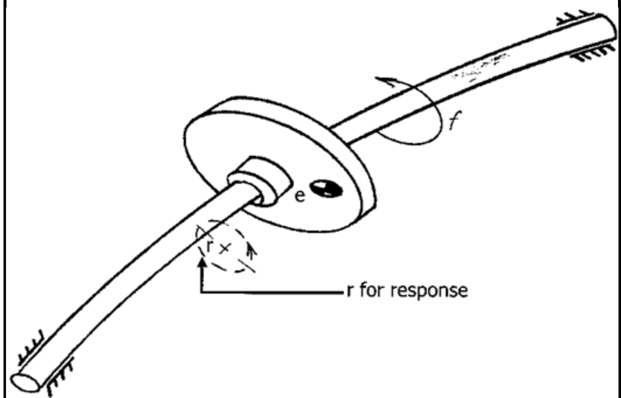
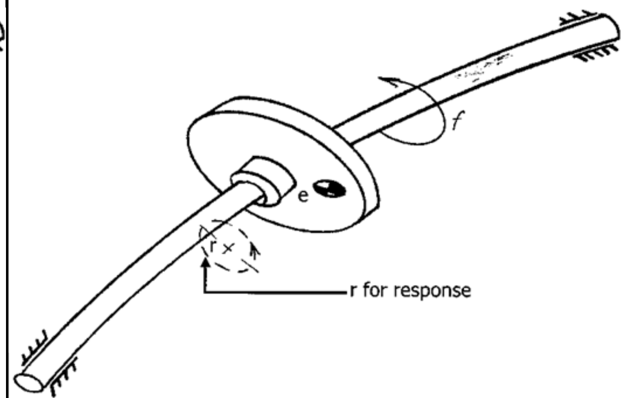
Int. Ref	C: MOTIONS	
❶ ❷ ❸ ❹	**Velocity (in/s)** $V = D(2\pi f)$ D = peak displacement, in f = frequency, cycle/s (cps or Hz)	**Velocity (mm/s)** $V = D(2\pi f)$ D = peak displacement, mm f = frequency, cycle/s (cps or Hz)

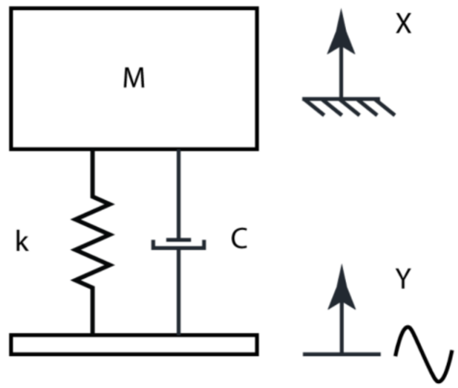
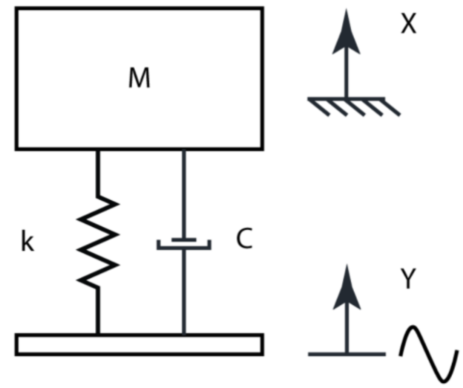
Int. Ref		
❶ ❷ ❸ ❹	**Acceleration** $A = V(2\pi f)$ A = acceleration, in/s^2 V = velocity, in/s f = frequency in Hz (CPS) *Note:* g_n = standard gravity, 386.1 in/s^2	**Acceleration** $A = V(2\pi f)$ A = acceleration, m/s^2 V = velocity, m/s f = frequency in Hz (CPS) *Note:* g_n = standard gravity, 9.807 m/s^2
Int. Ref	**D: FREQUENCIES**	
❷ ❸ ❹	**Bearing Frequencies*** $\mathrm{FTF} = \left(\frac{\Omega}{2}\right)\left[1 - \left(\frac{\mathrm{B}}{\mathrm{P}}\right)\cos(\mathrm{CA})\right]$ $\mathrm{BPFI} = \left(\frac{\mathrm{N}}{2}\right)\Omega\left[1 + \left(\frac{\mathrm{B}}{\mathrm{P}}\right)\cos(\mathrm{CA})\right]$ $\mathrm{BPFO} = \left(\frac{\mathrm{N}}{2}\right)\Omega\left[1 - \left(\frac{\mathrm{B}}{\mathrm{P}}\right)\cos(\mathrm{CA})\right]$ $\mathrm{BSF} = \left(\frac{\mathrm{P}}{2\mathrm{B}}\right)\Omega\left[1 - \left(\frac{\mathrm{B}}{\mathrm{P}}\right)^2\cos^2(\mathrm{CA})\right]$ FTF = fundamental train frequency BPFI = ball pass frequency, inner race BPFO = ball pass frequency, outer race BSF = ball spin frequency CA = contact angle, deg Ω = inner race speed (Hz or RPM) N = number of rolling elements per row P = pitch diameter, in B = ball or roller diameter, in Bearing defect frequency units correspond to inner race speed units *Rolling element bearings with rotating inner ring and stationary outer ring.	**Bearing Frequencies*** $\mathrm{FTF} = \left(\frac{\Omega}{2}\right)\left[1 - \left(\frac{\mathrm{B}}{\mathrm{P}}\right)\cos(\mathrm{CA})\right]$ $\mathrm{BPFI} = \left(\frac{\mathrm{N}}{2}\right)\Omega\left[1 + \left(\frac{\mathrm{B}}{\mathrm{P}}\right)\cos(\mathrm{CA})\right]$ $\mathrm{BPFO} = \left(\frac{\mathrm{N}}{2}\right)\Omega\left[1 - \left(\frac{\mathrm{B}}{\mathrm{P}}\right)\cos(\mathrm{CA})\right]$ $\mathrm{BSF} = \left(\frac{\mathrm{P}}{2\mathrm{B}}\right)\Omega\left[1 - \left(\frac{\mathrm{B}}{\mathrm{P}}\right)^2\cos^2(\mathrm{CA})\right]$ FTF = fundamental train frequency BPFI = ball pass frequency, inner race BPFO = ball pass frequency, outer race BSF = ball spin frequency CA = contact angle, deg Ω = inner race speed (Hz or RPM) N = number of rolling elements per row P = pitch diameter, mm B = ball or roller diameter, mm Bearing defect frequency units correspond to inner race speed units *Rolling element bearings with rotating inner ring and stationary outer ring.

❶ ❷ ❸ ❹	**General Guideline For Bearing Frequencies*** BPFO = 0.41 x N x Ω BPFI = 0.59 x N x Ω FTF = 0.41 x Ω BSF = 0.22 x N x Ω N = number of rolling elements per row Ω = speed units of RPM or Hz *for use in F_{Max} selection ONLY.	**General Guideline For Bearing Frequencies*** BPFO = 0.41 x N x Ω BPFI = 0.59 x N x Ω FTF = 0.41 x Ω BSF = 0.22 x N x Ω N = number of rolling elements per row Ω = speed units of RPM or Hz *for use in F_{Max} selection ONLY.
❷ ❸ ❹	**Natural Frequency – Classical** $f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ k = stiffness, lbf/in m = W/g_n W = weight, lbf g_n = standard gravity, 386.1 in/s^2 f_n = natural frequency of a single-degree-of-freedom system, Hz	**Natural Frequency - Classical** $f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ k = stiffness, N/m m = mass, kg f_n = natural frequency of a single-degree-of-freedom system, Hz
❸ ❹	**Natural Frequency - Static Deflection*** $f_n = \frac{1}{2\pi}\sqrt{\frac{g_n}{\delta_{st}}}$ g_n = standard gravity, 386.1 in/s^2 δ_{ST} = static deflection, in f_n = natural frequency of single-degree-of-freedom system, Hz *Only valid for linear spring rate and where the static deflection is in-line with gravity.	**Natural Frequency - Static Deflection*** $f_n = \frac{1}{2\pi}\sqrt{\frac{g_n}{\delta_{st}}}$ g_n = standard gravity, 9.807 m/s^2 δ_{ST} = static deflection, m f_n = natural frequency of single-degree-of-freedom system, Hz *Only valid for linear spring rate and where the static deflection is in-line with gravity.

❹	**Natural Frequencies – Undamped 2 Degree of Freedom** $\omega^2 = \frac{k_2 + k_1}{2m_2} + \frac{k_1}{2m_1} \pm \sqrt{\frac{1}{4}\left[\frac{k_2 + k_1}{m_2} + \frac{k_1}{m_1}\right]^2 - \frac{k_2 k_1}{m_2 m_1}}$ $f_{1,2} = \frac{1}{2\pi}\omega_{1,2}$  k_1, k_2 = stiffness, lbf/in m_1, m_2 = mass, lbf-s^2/in ω = natural frequency, rad/s	**Natural Frequencies – Undamped 2 Degree of Freedom** $\omega^2 = \frac{k_2 + k_1}{2m_2} + \frac{k_1}{2m_1} \pm \sqrt{\frac{1}{4}\left[\frac{k_2 + k_1}{m_2} + \frac{k_1}{m_1}\right]^2 - \frac{k_2 k_1}{m_2 m_1}}$ $f_{1,2} = \frac{1}{2\pi}\omega_{1,2}$  k_1, k_2 = stiffness, N/m m_1, m_2 = mass, kg ω = natural frequency, rad/s
❷ ❸ ❹	**Roll Rotational Speed** $\Omega = \frac{V}{5\pi D}$ V = web velocity, ft/min D = roll diameter, in Ω = rotational speed, RPS	**Roll Rotational Speed** $\Omega = \frac{16.66\,V}{\pi D}$ V = web velocity, m/min D = roll diameter, mm Ω = rotational speed, Hz
Int. Ref	**E: SIGNAL PROCESSING**	
❸ ❹	**Dynamic Range** $\mathrm{DR} = 20\log\frac{V_m}{V_r}$ $\frac{V_m}{V_r} = 10^{\frac{dB}{20}}$ V_m = voltage measured V_r = voltage reference DR = dynamic range, dB (decibels)	**Dynamic Range** $\mathrm{DR} = 20\log\frac{V_m}{V_r}$ $\frac{V_m}{V_r} = 10^{\frac{dB}{20}}$ V_m = voltage measured V_r = voltage reference DR = dynamic range, dB (decibels)
❶ ❷ ❸ ❹	**PEAK VS. RMS*** peak = 1.414 rms *Applies to harmonic waveforms only.	**PEAK VS. RMS*** peak = 1.414 rms *Applies to harmonic waveforms only.

❷ ❸ ❹	**Frequency Resolution*** Frequency resolution = (frequency span x window noise factor x 2) / # of FFT lines Window Noise Factor: 1.0 for uniform window 1.5 for Hanning window 3.8 for flat top window *ability to resolve closely spaced signals	**Frequency Resolution*** Frequency resolution = (frequency span x window noise factor x 2) / # of FFT lines Window Noise Factor: 1.0 for uniform window 1.5 for Hanning window 3.8 for flat top window *ability to resolve closely spaced signals
❷ ❸ ❹	**Data Acquisition Time (DAT)*** DAT = # FFT lines/frequency span *Applicable when Nyquist Factor = 2.56	**Data Acquisition Time (DAT)*** DAT = # FFT lines/frequency span *Applicable when Nyquist Factor = 2.56
❶ ❷ ❸ ❹	**Default Frequency Spans** Operating Speed $F_{MAX} \geq 10$ x RPM Rolling Element Bearings $F_{MAX} \geq 10$ x BPFI Fluid Film Bearings $F_{MAX} \geq 10$ x RPM Vane/Blade Pass $F_{MAX} \geq 3$ x # Vanes/Blades x RPM Electrical $F_{MAX} \geq 3$ x 2X Line Frequency Gear Mesh $F_{MAX} \geq 3$ x Gear Mesh Frequency RPM = rotational speed F_{MAX}= maximum frequency	**Default Frequency Spans** Operating Speed $F_{MAX} \geq 10$ x RPM Rolling Element Bearings $F_{MAX} \geq 10$ x BPFI Fluid Film Bearings $F_{MAX} \geq 10$ x RPM Vane/Blade Pass $F_{MAX} \geq 3$ x # Vanes/Blades x RPM Electrical $F_{MAX} \geq 3$ x 2X Line Frequency Gear Mesh $F_{MAX} \geq 3$ x Gear Mesh Frequency RPM = rotational speed F_{MAX}= maximum frequency

Int. Ref	F: VIBRATION RESPONSE	
❹	**Rotor Vibration Response**  $r = \frac{e\left(\frac{f}{f_n}\right)^2}{\sqrt{\left[1-\left(\frac{f}{f_n}\right)^2\right]^2+\left[2\zeta\frac{f}{f_n}\right]^2}}$ r = rotor vibration response, in peak e = eccentricity of mass, in f = forcing frequency, Hz f_n = natural frequency, Hz ζ = damping ratio, C/C_C C = damping constant, lbf s/in C_C = critical damping = 2 m ω_n m = mass, lbf s^2/in $\omega_n = \sqrt{\frac{k}{m}}$, rad/s	**Rotor Vibration Response**  $r = \frac{e\left(\frac{f}{f_n}\right)^2}{\sqrt{\left[1-\left(\frac{f}{f_n}\right)^2\right]^2+\left[2\zeta\frac{f}{f_n}\right]^2}}$ r = rotor vibration response, mm peak e = eccentricity of mass, mm f = forcing frequency, Hz f_n = natural frequency, Hz ζ = damping ratio, C/C_C C = damping constant, N s/m C_C = critical damping = 2 m ω_n m = mass, N s^2/m (= kg) $\omega_n = \sqrt{\frac{k}{m}}$, rad/s
❹	**Phase** $\tan\phi = \frac{2\zeta\frac{f}{f_n}}{1-\left(\frac{f}{f_n}\right)^2}$ ϕ = phase, deg f = forcing frequency, Hz f_n = natural frequency, Hz	**Phase** $\tan\phi = \frac{2\zeta\frac{f}{f_n}}{1-\left(\frac{f}{f_n}\right)^2}$ ϕ = phase, deg f = forcing frequency, Hz f_n = natural frequency, Hz

	Force Transmission/Base Motion	Force Transmission/Base Motion
❹	 $\frac{X}{Y}=\frac{F_{TR}}{F_0}=\frac{\sqrt{1+\left[2\zeta\frac{f}{f_n}\right]^2}}{\sqrt{\left[1-\left(\frac{f}{f_n}\right)^2\right]^2+\left[2\zeta\frac{f}{f_n}\right]^2}}$ F_{TR} = transmitted force, lbf F_0 = exciting force, lbf f = forcing frequency, Hz f_n = natural frequency, Hz ω_n = natural frequency, rad/s X = mass motion, in peak Y = base motion, in peak ζ = damping ratio = C/C_c k = spring stiffness, lbf/in C = damping constant, lbf s/in $C_c = 2m\omega_n$ = critical damping, lbf s/in m = mass, lbf-s^2/in	 $\frac{X}{Y}=\frac{F_{TR}}{F_0}=\frac{\sqrt{1+\left[2\zeta\frac{f}{f_n}\right]^2}}{\sqrt{\left[1-\left(\frac{f}{f_n}\right)^2\right]^2+\left[2\zeta\frac{f}{f_n}\right]^2}}$ F_{TR} = transmitted force, N F_0 = exciting force, N f = forcing frequency, Hz f_n = natural frequency, Hz ω_n = natural frequency, rad/s X = mass motion, mm peak Y = base motion, mm peak ζ = damping ratio = C/C_c k = spring stiffness, N/m C = damping constant, N s/m $C_c = 2m\omega_n$ = critical damping, N s/m m = mass, kg

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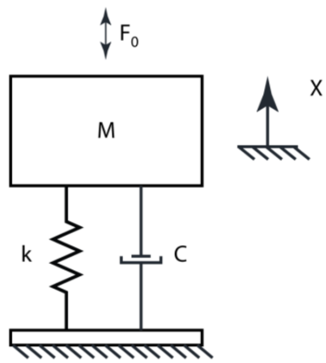
Forced Vibration Response



$$X = \frac{\frac{F_0}{k}}{\sqrt{\left[1-\left(\frac{f}{f_n}\right)^2\right]^2+\left[2\zeta\frac{f}{f_n}\right]^2}}$$

F_0 = exciting force, lbf
f = forcing frequency, Hz
f_n = natural frequency, Hz
ω_n = natural frequency, rad/s
X = mass motion, inches peak
ζ = damping ratio = C/C_c
k = spring stiffness, lbf/in
C = damping constant, lbf s/in
$C_c = 2M\omega_n$ = critical damping, lbf s/in
M = mass, lbf s^2/in

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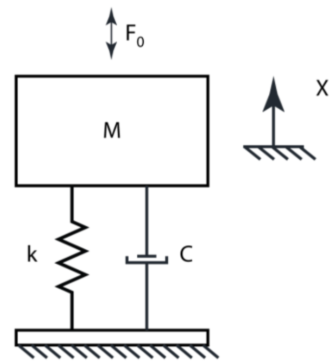
Forced Vibration Response



$$X = \frac{\frac{F_0}{k}}{\sqrt{\left[1-\left(\frac{f}{f_n}\right)^2\right]^2+\left[2\zeta\frac{f}{f_n}\right]^2}}$$

F_0 = exciting force, N
f = forcing frequency, Hz
f_n = natural frequency, Hz
ω_n = natural frequency, rad/s
X = mass motion, mm peak
ζ = damping ratio = C/C_c
k = spring stiffness, N/m
C = damping constant, N s/m
$C_c = 2M\omega_n$ = critical damping, N s/m
M = mass, kg

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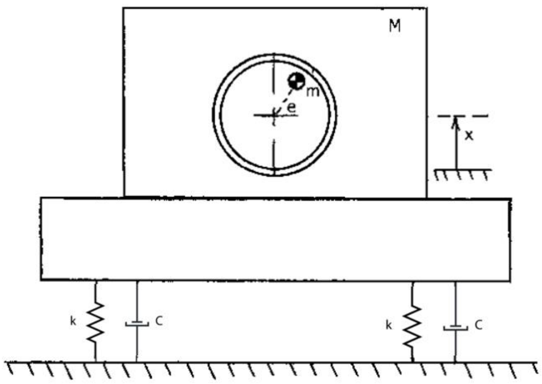
Mass Unbalance Induced Casing Response



$$\frac{MX}{me} = \frac{\left(\frac{f}{f_n}\right)^2}{\sqrt{\left[1 - \left(\frac{f}{f_n}\right)^2\right]^2 + \left[2\zeta \frac{f}{f_n}\right]^2}}$$

M = total mass, lbf s^2/in
m = rotor mass, lbf s^2/in
X = casing motion, inches peak
e = rotor eccentricity, inches
f = forcing frequency, Hz
f_n = natural frequency, Hz
ω_n = natural frequency, rad/s
ζ = damping ratio = C/C_c
k = spring stiffness, lbf/in
C = damping constant, lbf s/in
$C_c = 2M\omega_n$ = critical damping, lbf s/in

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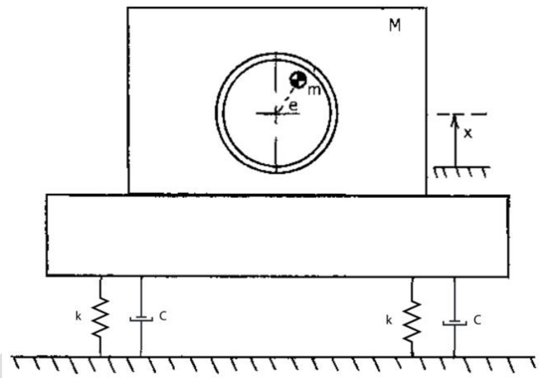
Mass Unbalance Induced Casing Response



$$\frac{MX}{me} = \frac{\left(\frac{f}{f_n}\right)^2}{\sqrt{\left[1 - \left(\frac{f}{f_n}\right)^2\right]^2 + \left[2\zeta \frac{f}{f_n}\right]^2}}$$

M = total mass, kg
m = rotor mass, kg
X = casing motion, mm peak
e = rotor eccentricity, mm
f = forcing frequency, Hz
f_n = natural frequency, Hz
ω_n = natural frequency, rad/s
ζ = damping ratio = C/C_c
k = spring stiffness, N/m
C = damping constant, N s/m
$C_c = 2M\omega_n$ = critical damping, N s/m

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G: STABILITY THRESHOLD

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Stability Threshold

$$\omega_s = \omega_o \sqrt{\frac{c}{g_n}}$$

ω_s = dimensionless stability threshold
ω_0 = journal operating speed, rad/s
c = bearing radial clearance, in
g_n = standard gravity, 386.1 in/s^2

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Stability Threshold

$$\omega_s = \omega_o \sqrt{\frac{c}{g_n}}$$

ω_s = dimensionless stability threshold
ω_0 = journal operating speed, rad/s
c = bearing radial clearance, m
g_n = standard gravity, 9.807 m/s^2

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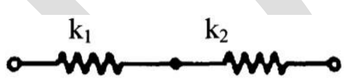
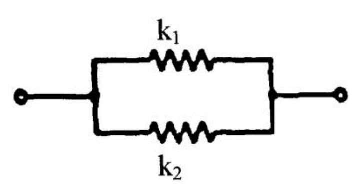
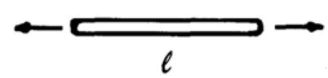
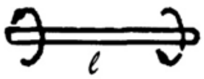
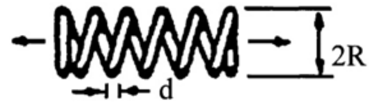
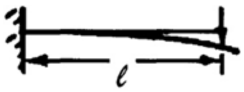
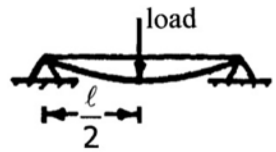
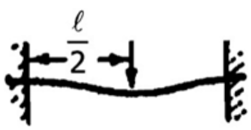
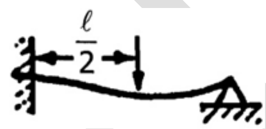
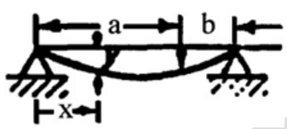
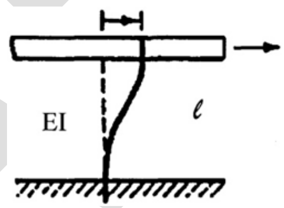
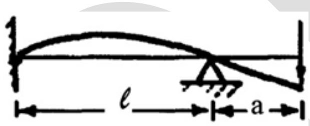
Int. Ref	**H: AMPLIFICATION FACTOR AND DAMPING RATIO**	
❹	**Half Power Method** $AF = \frac{N_{CR}}{N_2 - N_1}$ AF = amplification factor (at resonance) N_{CR} = critical speed, RPM N_2, N_1 = half power points, RPM	**Half Power Method** $AF = \frac{N_{CR}}{N_2 - N_1}$ AF = amplification factor (at resonance) N_{CR} = critical speed, Hz N_2, N_1 = half power points, Hz
❹	**Phase Change Method** $AF = \frac{\pi N_{CR}}{360} \frac{\Delta\phi}{\Delta N}$ AF = amplification factor (at resonance) N_{CR} = critical speed, RPM $\Delta\phi$ = phase change, degrees ΔN = speed change, RPM	**Phase Change Method** $AF = \frac{\pi N_{CR}}{360} \frac{\Delta\phi}{\Delta N}$ AF = amplification factor (at resonance) N_{CR} = critical speed, Hz $\Delta\phi$ = phase change, degrees ΔN = speed change, Hz
❹	**Time Domain Method** $\delta = \frac{1}{n} \ln \frac{X_0}{X_n}$ = log decrement $\zeta = \frac{C}{C_c} \approx \frac{\delta}{2\pi} \approx \frac{1}{2AF}$ = damping ratio n = number of cycles $\frac{X_0}{X_n}$ = amplitude ratio in n cycles C = damping constant, lbf s/in $C_C = 2M\omega_n$ = critical damping, lbf s/in AF = amplification factor M = system mass, lbf s^2/in ω_n = system natural frequency, rad/s	**Time Domain Method** $\delta = \frac{1}{n} \ln \frac{X_0}{X_n}$ = log decrement $\zeta = \frac{C}{C_c} \approx \frac{\delta}{2\pi} \approx \frac{1}{2AF}$ = damping ratio n = number of cycles $\frac{X_0}{X_n}$ = amplitude ratio in n cycles C = damping constant, N s/m $C_C = 2M\omega_n$ = critical damping, N s/m AF = amplification factor M = system mass, N s^2/m ω_n = system natural frequency, rad/s

❹	**Dual Constant/Dual Channel Method** Damping Calculation: $AF = \dfrac{\left(\frac{f_a}{f_b}\right)^2 + 1}{\left(\frac{f_a}{f_b}\right)^2 - 1}$ f_a = frequency above resonance, where the real or imaginary part of the transfer function reaches a peak f_b = frequency below resonance, where the real or imaginary part of the transfer function reaches a peak of opposite sign AF = amplification factor (at resonance) $AF \approx \dfrac{1}{2\frac{C}{C_c}} \approx \dfrac{1}{(2\zeta)}$ C = damping constant, lbf s/in $C_{C} = 2M\omega_n$ critical damping, lbf s/in AF = amplification factor M = system mass, lbf s^2/in ω_n = system natural frequency, $\frac{\text{rad}}{\text{s}}$	**Dual Constant/Dual Channel Method** Damping Calculation: $AF = \dfrac{\left(\frac{f_a}{f_b}\right)^2 + 1}{\left(\frac{f_a}{f_b}\right)^2 - 1}$ f_a = frequency above resonance, where the real or imaginary part of the transfer function reaches a peak f_b = frequency below resonance, where the real or imaginary part of the transfer function reaches a peak of opposite sign AF = amplification factor (at resonance) $AF \approx \dfrac{1}{2\frac{C}{C_c}} \approx \dfrac{1}{(2\zeta)}$ C = damping constant, N s/m $C_{C} = 2M\omega_n$ critical damping, N s/m AF = amplification factor M = system mass, N s^2/m ω_n = system natural frequency, $\frac{\text{rad}}{\text{s}}$
	I: MATERIAL PROPERTIES*	
❹	E = Modulus of Elasticity Aluminum 10.0 x 10^6 lbf/in^2 Ductile Iron 24.5 x 10^6 lbf/in^2 Steel 29.0 x 10^6 lbf/in^2 Titanium 14.9 x 10^6 lbf/in^2 G = Modulus of Rigidity Aluminum 3.9 x 10^6 lbf/in^2 Ductile Iron 9.3 x 10^6 lbf/in^2 Steel 11.2 x 10^6 lbf/in^2 Titanium 5.9 x 10^6 lbf/in^2	E = Modulus of Elasticity Aluminum 69 GPa Ductile Iron 169 GPa Steel 200 GPa Titanium 103 GPa G = Modulus of Rigidity Aluminum 27 GPa Ductile Iron 64 GPa Steel 77 GPa Titanium 41 GPa

	US Customary	SI
❹	Density** Aluminum 0.0978 lbm/in^3 Ductile Iron 0.2579 lbm/in^3 Steel 0.2836 lbm/in^3 Titanium 0.1620 lbm/in^3 *constants shown may be approximate, rounded, or stated for the field of vibration. **where gravity = g_n	Density Aluminum 2712 kg/m^3 Ductile Iron 7300 kg/m^3 Steel 7850 kg/m^3 Titanium 4500 kg/m^3 *constants shown may be approximate, rounded, or stated for the field of vibration. **where gravity = g_n
	J: Area Moments of Inertia	
❹	**Bending (Rectangular) Area Moment of Inertia** $I = \frac{\pi D^4}{64}$ (solid shaft) $I = \frac{\pi(D^4 - d^4)}{64}$ (hollow shaft) I = bending area moment of inertia of circular cross-section, in^4 D = outer diameter, in d = inner diameter, in	**Bending (Rectangular) Area Moment of Inertia** $I = \frac{\pi D^4}{64}$ (solid shaft) $I = \frac{\pi(D^4 - d^4)}{64}$ (hollow shaft) I = bending area moment of inertia of circular cross-section, mm^4 D = outer diameter, mm d = inner diameter, mm
❹	**Polar Area Moment of Inertia** $J = \frac{\pi D^4}{32}$ (solid shaft) $J = \frac{\pi(D^4 - d^4)}{32}$ (hollow shaft) J = polar area moment of inertia of circular cross section, in^4 D = outer diameter, in d = inner diameter, in	**Polar Area Moment of Inertia** $J = \frac{\pi D^4}{32}$ (solid shaft) $J = \frac{\pi(D^4 - d^4)}{32}$ (hollow shaft) J = polar area moment of inertia of circular cross section, mm^4 D = outer diameter, mm d = inner diameter, mm
	K: MASS MOMENTS OF INERTIA	
	Diametrical Mass Moment of Inertia $J_d = \frac{MD^2}{16}$ J_d = diametrical mass moment of inertia, lbf s^2 in M = mass, lbf s^2/in D = diameter, in	**Diametrical Mass Moment of Inertia** $J_d = \frac{MD^2}{16}$ J_d = diametrical mass moment of inertia, kg m^2 M = mass, kg D = diameter, mm

	Polar Mass Moment of Inertia $J_p = \dfrac{MD^2}{8}$ J_p = diametrical mass moment of inertia, lbf s^2 in M = mass, lbf s^2/in D = diameter, in	**Polar Mass Moment of Inertia** $J_p = \dfrac{MD^2}{8}$ J_p = diametrical mass moment of inertia, kg m^2 M = mass, kg D = diameter, mm
	L: CASES OF STIFFNESS CALCULATION **All cases shown apply to both US Customary and SI systems**	
❹	**Common Parameters for US Customary Usage of Case formulas** k = stiffness, lbf/in k_t = torsional stiffness, in lbf/rad I = bending area moment of inertia of circular cross-section, in^4 J = polar area moment of inertia of circular cross section, in^4 D = outer diameter, in d = inner diameter, in d = rod diameter of coil, in A = cross sectional area, in^2 n = number of turns, count *l* = total length, in E = Modulus of Elasticity, lbf/in^2 G = Modulus of Rigidity, lbf/in^2	**Common Parameters for SI Usage of Case formulas** k = stiffness, N/m k_t = torsional stiffness, N m/rad I = bending area moment of inertia of circular cross-section, m^4 J = polar area moment of inertia of circular cross section, m^4 D = outer diameter, m d = inner diameter, m d = rod diameter of coil, m A = cross sectional area, m^2 n = number of turns, count *l* = total length, m E = Modulus of Elasticity, Pa G = Modulus of Rigidity, Pa GPa = 10^9 Pa
❹	**Case A: Springs in series**  $k = \dfrac{1}{1/k_1 + 1/k_2}$	**Case B: Springs In parallel**  $k = k_1 + k_2$
❹	**Case C:**  $k = \dfrac{EI}{\ell}$	**Case D:**  $k = \dfrac{EA}{\ell}$

❹	**Case E:**  $k = \frac{GJ}{\ell}$	**Case F:**  $k = \frac{Gd^4}{64nR^3}$
❹	**Case G:**  $k = \frac{3EI}{\ell^3}$	**Case H:**  $k = \frac{48EI}{\ell^3}$
❹	**Case I:**  $k = \frac{192EI}{\ell^3}$	**Case J:**  $k = \frac{768EI}{7\ell^3}$
❹	**Case K:**  $k = \frac{3EI\ell}{a^2b^2}$ $\ell = a + b$	**Case L:**  $k = \frac{12EI}{\ell^3}$
❹	**Case M:**  $k = \frac{3EI}{(\ell + a)a^2}$	**Case N:**  $k = \frac{24EI}{a^2(6\ell + 8a)}$

Instructional / Example
"Bubble" style answer sheet